Symmetry Plane Laminar and Turbulent Viscous Flow on Bodies at Incidence

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Theme

A METHOD is developed for solving the thin viscous shock-layer equations for flow along the symmetry plane of sharp or blunt bodies at angle of attack in hypersonic flow. The governing equations are obtained from the Navier-Stokes equations by retaining terms up through second order in the inverse square root of the Reynolds number. The resulting equations are solved using a Crank-Nicolson type finite difference scheme.

Contents

The procedure generally used for this type of problem at zero incidence involves solving the uncoupled inviscid and boundary-layer equations and coupling the solution in an iterative manner. Because of difficulties involved in matching the boundary layer with the inviscid flow for angle-of-attack problems, this method cannot be readily extended. In the present approach, the complete shock-layer flow is obtained from a single set of equations developed from the general steady-state Navier-Stokes equations using effective transport parameters. The matching problem is therefore eliminated, and displacement thickness effects on inviscid flow are included within the shock-layer approximation.

Davis[1] developed this concept for laminar flow over bodies at zero angle of attack. The method was extended by Eaton and Kaestner[2] to the case of laminar flow on the windward planes of sharp cones at angles of attack.

The problem is further extended in this paper[3] to include combinations of sharp or blunt bodies at angle of attack in laminar or turbulent flow. The analysis is developed for arbitrarily shaped bodies with a minimum of one plane of symmetry. The flow between the bow shock and body surface is calculated by solving the shock-layer equations. Nonlinear terms are locally linearized. The resulting parabolic equations are tridiagonal in form and are solved using the Thomas algorithm.[4] To address the turbulent problem, a mean velocity closure scheme is incorporated into the method by using a "Van Driest" type two-layer mixing length model with exponential damping near the wall. Properties at the shock boundary are calculated using Rankine-Hugoniot equations. The body surface temperature is specified and the surface velocities are set equal to zero. The resulting computer code (VIS) written for the CDC 6600 accommodates blunt or sharp power-law bodies, hyperboloids, and cones with elliptical cross sections. However, results are only presented[3] for bodies with axisymmetric cross sections.

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Calculations were made on a 5° half-angle cone in air at $M_\infty = 8$, $p_\infty = 570$ pascals, and $\rho_\infty = 0.036$ Kg/m^3. The free-stream Reynolds number of 1.1×10^7/m results in transition between $x = 25$ and $x = 55$ cm from the cone tip. These are the experimental conditions used by Martellucci et al.,[5] in the AEDC von Kármán Facility. Using this experimentally located transitional region as input conditions to the VIS program, the complete windward-plane flowfield along the cone was calculated. At each downstream station (x) the edge conditions used in the eddy viscosity model are those properties located where the tangential velocity equals 95% of that at the shock. The boundary-layer thickness for these runs is typically less than 10% of the shock layer. Therefore, Figs. 1 and 2 show only that portion of the shock layer. A variable-mesh spacing consisting of 100 mesh points is used through the shock layer and 40 of these points are located in the boundary layer.

Figure 2 shows a comparison between experimentally obtained velocity and temperature data and VIS code calculations for angle of incidence $\alpha = 0$, 3, and 5°. It was found that to obtain convergence through the transition region, the Δx step size must be reduced to 0.15 cm. This is one order of magnitude smaller than that generally required for laminar flow on a cone. A total of 850 sec of computer time is required to calculate 1 m along the body. The trends of the calculated and experimental boundary-layer profiles agree at all angles of incidence. Both methods show a similar decrease in boundary-layer thickness as the angle of attack is increased.

To obtain better agreement with experiment, the coefficients used in the turbulent viscosity model were parametrically varied.

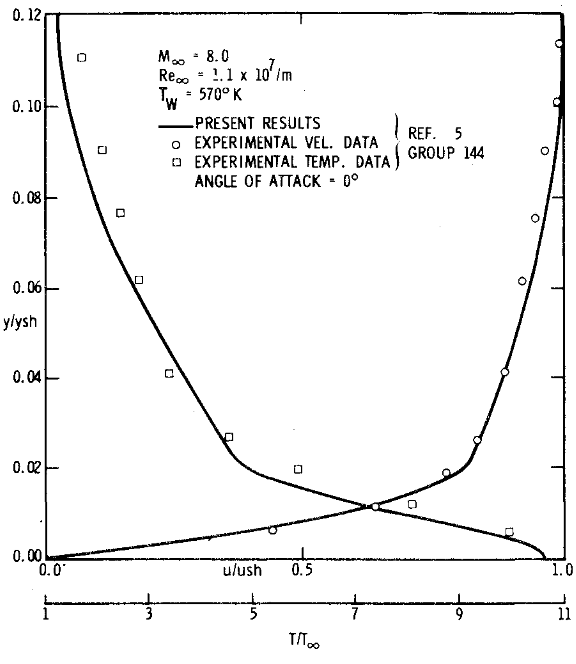
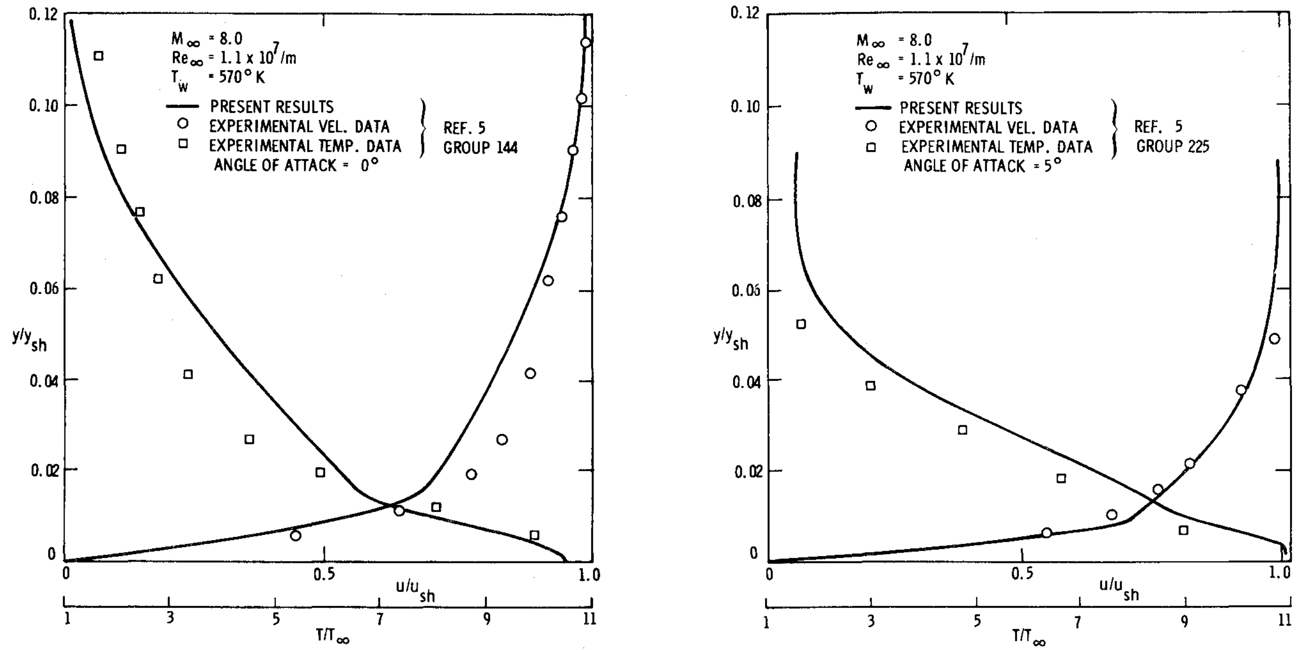


Fig. 1 Velocity and temperature distribution in a turbulent boundary layer on a sharp cone at $x = 73$ cm, $\alpha = 0$, perturbed eddy viscosity model.



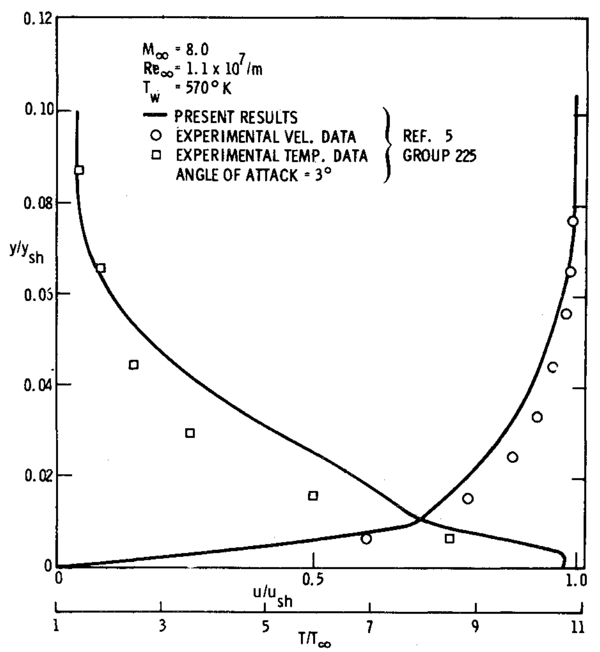


Fig. 2 Velocity and temperature distribution in a turbulent boundary layer on a sharp cone at $x = 73$ cm.

It was found that good agreement could be obtained for $\alpha = 0$ (Fig. 1). However, when this revised model was used for the $\alpha = 3$ and 5 case, the results show no improvement over the original model. It is felt that a refinement to the currently used turbulent model should be the addition of the density fluctuation terms. This compressibility effect is discussed by Shang.[6] He has shown that it is significant in the Mach number range being considered.

In summary, the laminar shock-layer technique previously developed[2] for cones at angles of attack has been extended to include: 1) a mean velocity closure scheme for turbulent flows; and 2) capability of handling shapes which are not bodies of revolution. The method is shown to be well suited for the shock-layer problem over a wide range of Reynolds numbers in both the laminar and turbulent regime.

References

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